Five Year Integrated M.Sc. Examination, 2021-2022
Semester - V
Code: PH-3-5-5

Time: Three Hours | Subject: Statistical Mechanics | Full Marks: 40

Questions are of values as indicated in the margin
Answer question number **one** and any **three** from the rest

1. Answer **any five** questions

$5 \times 2 = 10$

(a) Define Microcanonical ensemble.

(b) Briefly explain the principle of detailed balance.

(c) Draw the phase space diagram of a particle with energy E which is otherwise free but only allowed to move in a straight line extended from $-L$ to L.

(d) Show that the canonical partition function for a non-interacting two particles system can be written as a square of single particle partition function.

(e) Consider a system of ten boxes. How many way one can distribute eight balls among these boxes? Explain your answer.

(f) What is the dimension of phase space of five rigid triatomic molecules? Explain your answer.

(g) State and explain the postulate of equal a priori probability.

2. (a) Consider a 1D box of size L centred on $x = 0$. Say there are **two** identical particles in the box and the probability function of finding each is $P(x)$.

i. If x is the position of centre-of-mass ($x = \frac{x_1+x_2}{2}$), calculate the expectations $\langle x \rangle$ and $\langle x^2 \rangle$.

ii. Show that the standard deviation of the centre-of-mass has shrunk by a factor of $\sqrt{2}$ from the one particle case for any $P(x)$.

(b) Consider a game in which **two** true dice are rolled. Find the probability of obtaining

i. exactly one ace

ii. at least one ace

iii. exactly two aces

(3+2)+5=10

3. (a) A particle performs random walk motion about the origin O, and at each step it moves with equal probability by a unit distance either to the left or to the right. Calculate the standard deviation for such random walk after the particle takes total N number of steps.

   (b) Consider a large container filled with O_2 gas. Due to some manufacturing defect there is a leak in the container and the O_2 gas comes out through this leak and gradually diffuses into atmosphere. Treating this diffusion problem as a random walk, one can calculate the probability distribution $P_t(x)$ for where a O_2 molecule is after a time t. At large time limit ($t \to \infty$), one can write

$$P_t(x) = \sqrt{\frac{1}{2\pi l \bar{v} t}} \exp\left[-\frac{x^2}{2tl\bar{v}}\right] ,$$

   where $l =$ the mean free path, $\bar{v} =$ the average molecular speed. Hence show that $P_t(x)$ satisfy 1D diffusion equation.

5+5=10

4. (a) The energy levels accessible to a molecule have energy $E_1 = 0$ and $E_2 = \Delta$ (where Δ is a constant). A gas of these molecules is in thermal equilibrium at temperature T. Calculate the average energy $\bar{E}$ and the specific heat at constant volume C_v. Show that

   (b) In a thermodynamic system in equilibrium each molecule can exist in three possible states with probabilities $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{6}$. Calculate the entropy per molecule.

(3+3)+4=10

5. (a) Calculate the number of accessible states $\Omega(E, N, V)$ to an ideal gas with total energy E and consists of N gas molecules. Moreover, this gas is confined within a thermally insulated container of volume V.

   (b) Using starling approximation calculate the entropy of the ideal gas. Show that this expression for entropy leads to Gibbs' paradox.

   (c) How do you resolve this paradox ? Write the correct expression for entropy of an ideal gas.

3+4+3=10

6. (a) Consider an isolated system A^0 which is divided into two sub systems A and A'. Since $A^0 \equiv A + A'$ is isolated, its energy $E^0 = E + E'$ is constant. Sub systems A and A' can only exchange energy among themselves. Establish the condition for thermal equilibrium between A and A'.

   (b) Derive the partition function for canonical ensemble. Clearly state the assumptions that one needs to adopt to derive canonical partition function.

   (c) Stating from the canonical partition function show that the energy fluctuation (ΔE) is proportional the heat capacity C_V.

3+3+4=10